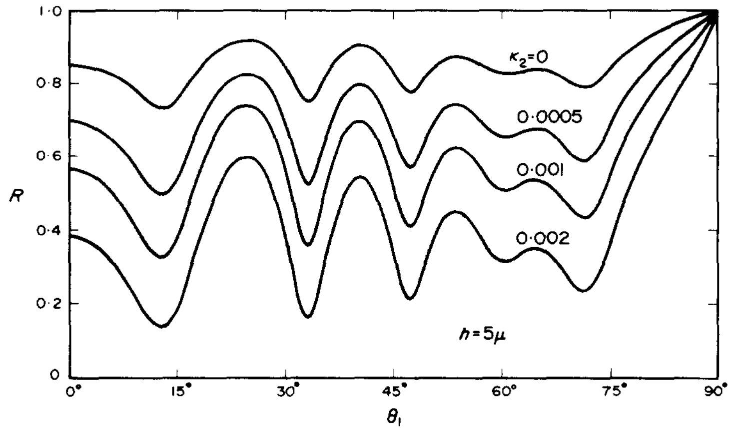

FIG. 3. Effect of κ_2 on the reflectivity, $h = 5\ \mu$.

film-substrate system but with $\kappa_2 = 0$. Furthermore, it is additionally assumed that the $\theta_{1(1)}$ and $\theta_{1(2)}$ can be associated with the angular positions of tangencies between the $\kappa_2 = 0$ reflectivity curve and the envelope curves.

With these postulates, the calculation procedure outlined in [1] for perpendicular polarized radiation can now be employed for parallel polarized radiation. In the execution of the method, equation (10) of the reference is replaced by equation (5) with ϕ_{23} from equation (1). The assignment of even or odd values to m is based on the rules stated in the text following equation (4).

Experiments described in [1] have established the practical utility of these methods for determining the optical constants and thickness of thin films.

ACKNOWLEDGEMENT

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ON A SIMPLE CORRELATION FOR PRANDTL NUMBER EFFECT ON FORCED CONVECTIVE HEAT TRANSFER WITH SECONDARY FLOW

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1. INTRODUCTION

THE PRANDTL number effect on fully developed laminar forced convection heat transfer with secondary flow was studied recently for uniformly heated horizontal tubes [1] and curved rectangular channels [2]. A careful examination of the energy equation and the numerical results reveals that when the Prandtl number is large the inertia terms in the momentum equations can be neglected. The role of Prandtl number in the advective terms is seen to be equivalent to the characteristic parameter $ReRa$ for uniformly heated horizontal tubes [1] or Dean number K for curved channels [2]. The above observation leads to the possibility of numerical solution for Graetz problem in straight or curved tubes or channels with significant secondary flow effect for large Prandtl number fluids. In this formulation, the advective terms in the energy equation must be retained because of large Prandtl number, and the weak secondary flow is seen to play an important role through the advective terms.

2. RESULTS

A study of heat transfer results in the form of $Nu/(Nu)_0$ vs. $ReRa$ [1] and $Nu/(Nu)_0$ vs. K [2] shows that after reaching a certain parametric value, an asymptotic behavior appears for each Prandtl number fluid, and furthermore all the curves for $Pr \geqslant 1$ are parallel to each other suggesting that a new correlation for Prandtl number effect is possible by using a new parameter.

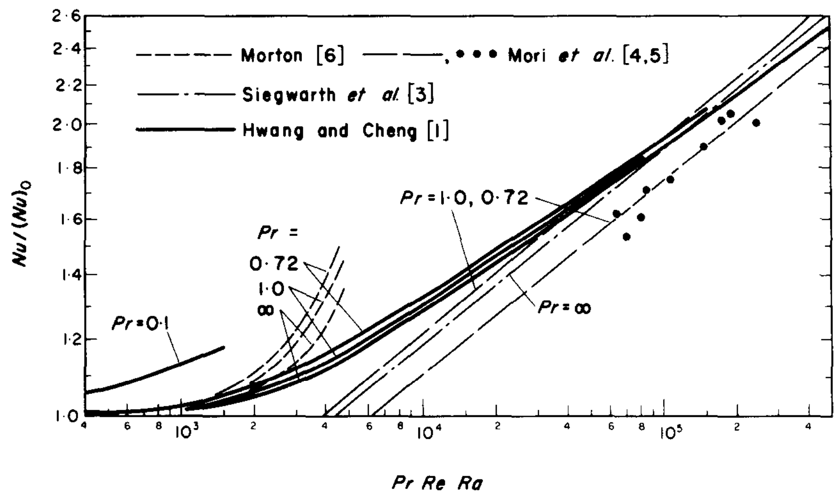

FIG. 1. A new correlation for Prandtl number effect on combined free and forced laminar convection heat transfer in horizontal tubes based on $Nu/(Nu)_0$ vs. $PrReRa$.

All the heat transfer results shown in Fig. 7 of [1] are replotted in Fig. 1 on the basis of $Nu/(Nu)_0$ vs. a new parameter $PrReRa$. The curve corresponding to the correlation $N = 0{\cdot}471\ (GP)^{1/4}$ given by Siegwarth *et al.* [3] for $Pr \to \infty$ is also shown after transforming into $Nu/(Nu)_0 = 0{\cdot}190\ (PrReRa)^{1/4}$ by using the relations $GP = PrReRa/(2Nu)$. $N = Nu/2$ and $(Nu)_0 = 4{\cdot}36$. It is noted that Mori and Futagami's solution [4] using boundary layer approximation deviates by about 1·5 per cent only from the above curve. Furthermore, the theoretical and experimental results by Mori *et al.* [4, 5] for $Pr = 0{\cdot}72$ contradict with both the numerical solution [1] and the perturbation solution [6]. For the same value of $PrReRa$, the numerical and perturbation solutions predict higher heat transfer rate for low Prandtl number fluids. It is significant that with the new correlation the numerical solution for $Pr = O(1)$ already approaches the asymptotic solution for $Pr \to \infty$. The correlation given by Siegwarth *et al.* [3] for $Pr \to \infty$ is also seen to coincide with the new correlation curve for $Pr \to \infty$ after reaching $PrReRa = 3 \times 10^4$. Note the insignificant difference between the numerical results for $Pr = 1$ and ∞. This observation is believed to be significant since separate experiment or numerical solution is not required for various Prandtl numbers ranging from $O(1)$ to ∞. It should be pointed out that the present new correlation is valid only within the range of the numerical study [1] and the correlation equation given by Siegwarth *et al.* [3] for $Pr \to \infty$ is believed to be still valid outside the range of $PrReRa$ shown in Fig. 1. One should also note that the numerical study has difficulty at higher values of $PrReRa$ and this may account for the fact that the slope of the new correlation is not the same as that obtained from the data in [3, 5] when $PrReRa > 2 \times 10^5$.

Similarly, all the heat transfer results for $Pr = 0{\cdot}71 \sim 10^4$ shown in Fig. 17 of [2] for curved square channel are

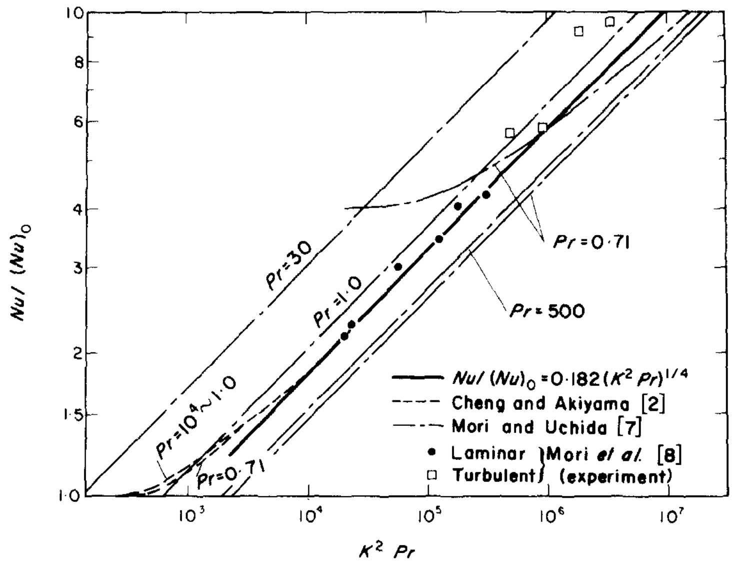

FIG. 2. A new correlation for Prandtl number effect on laminar forced convection heat transfer in curved square channel based on $Nu/(Nu)_0$ vs K^2Pr.

replotted in Fig. 2 on the basis of $Nu/(Nu)_0$ vs. a new parameter K^2Pr with comparison made against Mori and Uchida's analytical solution [7] based on boundary-layer approximation and experimental data by Mori *et al.* [8]. It is again seen that with a new parameter K^2Pr all the heat transfer results for $Pr = 1{\cdot}0 \sim \infty$ coincide, and only a slight deviation is observed for $Pr = 0{\cdot}7$ in low parameter range. The new correlation equation $Nu/(Nu)_0 = 0{\cdot}182 (K^2Pr)^{\frac{1}{4}}$ for curved square channel is obtained by linear extrapolation of the numerical solution based on the asymptotic behavior and is valid for $1 \leqslant Pr \leqslant 10^4$ and $Nu/(Nu)_0 \geqslant 1{\cdot}5$. The new correlation curve agrees with the experimental data [8] better than the analytical result from boundary-layer approximation [7] confirming the effectiveness of the new correlation. Figure 2 also shows that boundary-layer approximation predicts inconsistent Prandtl number effect.

3. CONCLUDING REMARKS

The purpose of this communication is to point out a simple correlation for Prandtl number effect on fully developed laminar forced convective heat transfer with secondary flow caused by such body forces as buoyancy and centrifugal forces based on an asymptotic behavior for $Pr \to \infty$. The Prandtl number effect for laminar forced convective heat transfer is clarified, and the extension to cases with other body forces is obvious. The clarification of the Prandtl number effect is believed to be significant since Graetz problem with significant secondary flow effect can now be approached by numerical method.

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EXPERIMENTAL OBSERVATIONS OF WAKE FORMATION OVER CYLINDRICAL SURFACES IN NATURAL CONVECTION FLOWS

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INTRODUCTION

NATURAL convection flows generated adjacent to horizontal cylinders, spheres and other submerged shapes has received considerable attention in the past. However, most studies concern the nature of the portions of such flows attached to the surface. These may be described by simplified mathematical models, such as boundary layer theory. The subsequent regions, where the flow may separate from the surface and rise perhaps as a buoyant plume, has not been considered in detail. The mathematical difficulties are extreme. apparently nothing is known concerning the possible occurrence in natural convection flows of the analogue of separation in forced flows. There have been no studies of these regions or demonstration of the flow reversal commonly observed in forced flows, although such terminology has often been used in describing natural convection flows. However, in such flow the mechanism must be very much more complicated. There is no external mechanism driving separation as does the pressure gradient in forced flows. This separates the flow. In natural convection the principal driving mechanism is the body force which operates only near the surface.

The experiment described here was an attempt to observe the nature of thermally induced convection flow on the upper side of a cylindrical surface, to determine some of its characteristics, and to study any flow separation and reversal near the surface. We are concerned with sufficiently vigorous flows so that the boundary-layer regime is found upstream. It is well known that similar boundary layers form on the opposite sides of a heated cylinder placed horizontally in an extensive fluid. The fluid in each of these layers generated by buoyancy forces will rise and move around the cylindrical surface toward the top. Near and at the top they merge to form a two-dimensional plume which rises above the cylinder. The present experiment was designed to attempt to observe the mechanism of this interaction and to detect any separation and secondary flows which may be generated.

Most previous studies of natural convection flow around spherical and cylindrical objects were concerned with either attached flows or only with overall transport characteristics. Merk and Prins [1] (1954) presented analytical results of the heat transfer around horizontal cylinders and spheres. They estimated the location of the region where the boundary layer was said to separate from the variation of the heat transfer around the heated surface. Bromhan and Mayhew [2] (1961) presented an experimental correlation of the heat transfer around spheres in air and also observed from smoke tests flow separation near the upper section of the sphere. The observed separation point depended on the location of smoke injection. Thus, there is some question of the meaning of these observations.

Kranse and Schenk [3] (1965) studied the natural convection from spheres by melting solid benzene spheres in liquid benzene. The local Nusselt number around the sphere was measured by recording the rate of decrease of the sphere's diameter. Schenk and Schenkels [4] (1968) carried out a similar experiment with melting spheres of ice in water. It was observed that when a positive thermal expan-